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ECONOMICS RELATED TO EXPLOITATION OF GAS WELLS AT CONSTANT FLOW

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1 INTRODUCTION

The problem of gas flow through porous mediums with application to exploitation of fields shows serious difficulties even in the simple case of plane-quaquaversal flow through a homogenous and isotropic porous medium. Indeed, even if it accepted that the flowing phenomenon is usually isothermal, the pressure differential equation is non-linear and as for the gas behaviour both the deviation from the perfect gases low and the viscosity pressure variation should be taken into account.

The mathematical model described in this study takes into account all the above aspects, therefore the use of a numerical method of solving is required. Such method is applied in the case of a permeable field exploited at various rates of flow. For each of these the variation of gas pressure at top of the well is to be determined.

2 MATHEMATICAL MODEL

The process of gas plane-quaquaversal isothermal flow through a homogenous and isotropic porous medium is simulated by equation [1]

$$\nabla\left(\frac{p}{\mu Z}\nabla p\right)=\frac{m}{k}\frac{\partial}{\partial t}\left(\frac{p}{Z}\right) \qquad (1)$$

where ∇-Hamiltonian, m and k – porosity, i.e. the permeability of porous medium(constants), p – field gases pressure, and μ and Z – dynamic viscosity, i.e. the factor of gas deviation from the perfect gas model, both depending on pressure. Equation (1) may also be written as

$$\frac{\partial^2 p}{\partial p^2}+\frac{1}{r}\frac{\partial p}{\partial r}-\frac{\psi}{\varphi}\frac{\partial p}{\partial t}=-\frac{1}{\varphi}\frac{d\varphi}{dp}\left(\frac{\partial p}{\partial r}\right)^2 \qquad (2)$$

where the function φ and ψ result from the relations

$$\varphi=\frac{p}{Z\mu};\ \psi=\frac{m}{k}\frac{1}{z^2}\left(Z-p\frac{dZ}{dp}\right) \qquad (3)$$

and depend on gas pressure only. Equation (2) represents the mathematical model of the process of gas flow through a circular field towards a central well, its solution providing the gas pressure distribution in time according to radius. Introducing a new dimensionless variable by

$$\xi=\frac{\ln r}{\ln \overline{R_c}};\ \bar{r}=\frac{r}{R_s};\ \overline{R_c}=\frac{R_c}{R_s} \qquad (4)$$

R_s being the well radius and R_c the field outline, as well as the low pressure $P(\xi,t)$ by

$$P(\xi,t)=\frac{p(\xi,t)}{p_c} \qquad (5)$$

p_c being the critical pressure, equation (2) becomes [2]

$$\frac{\partial^2 P}{\partial \xi^2}-a\left(\overline{R_c}\right)^{2\xi}\Phi\frac{\partial P}{\partial t}=-b\left(\frac{\partial P}{\partial \xi}\right)^2\Psi \qquad (6)$$

where the functions Φ and Ψ result from the expressions

$$\Phi(P) = \mu\left(\frac{1}{P} - \frac{1}{Z}\frac{\mathrm{d}Z}{\mathrm{d}P}\right); \qquad (7)$$

$$\Psi(P) = \left(\frac{1}{P} - \frac{1}{Z}\frac{\mathrm{d}Z}{\mathrm{d}P} - \frac{1}{\mu}\frac{\mathrm{d}\mu}{\mathrm{d}P}\right), \qquad (8)$$

and the coefficients *a* and *b* are calculated with

$$a = \frac{m\mu_0}{kp_c} R_s^2 \ln^2\left(\overline{R_c}\right);\ b = \frac{1}{8p_c}. \qquad (9)$$

Equation (6) simulating the process of gas flow towards a central well into a homogenous and isotropic circular field is a differential equation with partial derivative of 2^{nd} degree of parabolic type and non-linear. It admits no analytic solution, but a computer assisted numerical approach.

3 INITIAL AND LIMIT CONDITIONS

The initial condition transposes mathematically the fact that the gas pressure in entire field has the value p_z at the initial moment, i.e.

$$P(\xi, 0) = P_z = \frac{P_z}{P_c} \qquad (10)$$

The limit conditions are necessary in terms of field exploitation method and also of its characteristic. Hence, in case the field is exploited at constant rate of flow, the condition of maintaining constant the gas velocity at well outlet is to be set according to the well exploitation flow:

$$\left.\frac{\partial P}{\partial \xi}\right|_{\xi=0} = \frac{Q_s}{A}, \qquad (11)$$

Q_s being the gas rate of flow, and *A* a constant resulted from

$$A = \frac{2\pi H R_s k p_s p_c T_N}{Z_s \mu_s p_N T_s R_c} \qquad (12)$$

where *H* – thickness of carrier bed, p_s – gas pressure at top of the well, T_s – gas temperature at top of the well, μ_s – dynamic viscosity corresponding to this pressure, Z_s – compresibility factor corresponding to this pressure, p_N – normal pressure, and T_N – normal temperature.

The second limit condition refers to the field outline. Thus, in the case of closed field with permeable outline(with water pushing), the condition of maintaining constant the pressure p_z is necessary on such outline

$$P(1, t) = P_z = \frac{p_z}{p_c} \qquad (13)$$

and in the case of closed field with impermeable outline, the condition of maintaining zero flow on such outline is necessary, i.e.

$$\left.\frac{\partial P}{\partial \xi}\right|_{\xi=1} = 0 \qquad (14)$$

4 NUMERICAL METHOD

We will transform the continuous spectrum *C:* $[0 \le \xi \le 1,\ 0 \le t \le T]$ into the point discrete lattice R_{ij} :[$\xi=(i\text{-}1)\cdot h$, $t=j\cdot\tau_i n_i$], where $i=1\div n$, $j=0\div m$ are the spatial index and the temporal index, respectively, and h and τ are the spatial pitch and temporal pitch, respectively, and n and m are their numbers. Thus, instead of exact values of pressure $P(\xi, t)$ we will consider the discrete approximate values $P_i^j = P\left(\xi_i, t_j\right)$.

Making use of calculus scheme with finite quotients of Hyman Kaplan implicit type [3], known as stable unconditionally and absolutely convergent, as well as end approximations proposed by West, Garvin and Sheldon [2], equation (8) becomes a system of equations generated by the scheme

$$P_{i-1}^{j+1} - \left(2 + C_i^{j+1}\right) P_i^{j+1} + P_{i+1}^{j+1} = -C_i^{j+1} P_i^j - D_i^{j+1} \tag{15}$$

for i = 2÷n, where

$$C_i^{j+1} = \frac{ah^2}{\tau}\left(\overline{R_c}\right)^{2h(i-1)} \Phi\left(P_i^{j+1}\right) \tag{16}$$

$$D_i^{j+1} = \frac{b}{4}\Psi\left(P_i^{j+1}\right)\left(P_{i+1}^{j+1} - P_{i-1}^{j+1}\right)^2 \tag{17}$$

$$\Phi\left(P_i^{j+1}\right) = V_i^{j+1}\left(\frac{1}{P_i^{j+1}} - \frac{Zp_i^{j+1}}{Z_i^{j+1}}\right) \tag{18}$$

$$\Psi\left(P_i^{j+1}\right) = \left(\frac{1}{P_i^{j+1}} - \frac{Zp_i^{j+1}}{Z_i^{j+1}} - \frac{Vp_i^{j+1}}{V_i^{j+1}}\right) \tag{19}$$

$$V_i^{j+1} = \mu\left(P_i^{j+1}\right);\ Vp_i^{j+1} = \frac{\mathrm{d}\mu}{\mathrm{d}P}\left(P_i^{j+1}\right), \tag{20}$$

$$Z_i^{j+1} = Z\left(P_i^{j+1}\right);\ Zp_i^{j+1} = \frac{\mathrm{d}Z}{\mathrm{d}P}\left(P_i^{j+1}\right). \tag{21}$$

completed with

$$P_i^j = Pz\,;\ P_{n+1}^{j+1} = Pz \tag{22}$$

$$-3P_1^{j+1} + 4P_2^{j+1} - P_3^{j+1} = \frac{2hQ}{A} \tag{23}$$

$$3P_{n-1}^{j+1} - 4P_n^{j+1} + P_{n+1}^{j+1} = 0 \tag{24}$$

Here the values of low pressure at temporal level j are considered as known, and unknown at temporal level $j+1$, respectively.

5 NUMERICAL SIMULATION OF FIELD EXPLOITATION

In order to solve the system of linear algebraic equations generated by the scheme with finite quotients (15) we will mark with $P0(i)$ the distribution of low pressure corresponding to known moment, j, and with $P(i)$ the distribution of low pressure corresponding to unknown moment, $j+1$. The two approximate solutions successively obtained by solving the algebraic system generated by the calculus scheme adopted will be marked with $P1(i)$ and $P2(i)$, respectively; obviously i will take values from 1 to $n+1$.

To facilitate the compiling of the calculus program we will define the supporting functions $Z(X)$, $Zp(X)$, $V(X)$, $Vp(X)$ that will help us calculate the value sets of deviation factor $Z(i)$, $Zp(i)$ and dynamic viscosity $V(i)$, $Vp(i)$, respectively, corresponding to the distribution of low pressure at moments required by running the calculus program, i.e. for $P0(i)$, $P1(i)$or $P2(i)$. Now the values corresponding to expressions $C(i)$ and $D(i)$ can be calculated based on calculus procedures.

The conditions (22), (23) and (24), respectively are obviously written as

$$P0(i) = Pz\,;\; P(1) = Ps;\; P(n+1) = Pz \tag{25}$$

$$-3P(1) + 4P(2) - P(3) = \frac{2hQ}{A} \tag{26}$$

$$3P(n-1) - 4P(n) + P(n+1) = 0 \tag{27}$$

In the case of permeable closed field exploited at constant pressure, the calculus scheme (15)

With the conditions (25) generate the following system of equations

$$P(1) = Ps \tag{28}$$

$$P(i-1) - \left[2 + C(i)\right]P(i) + P(i+1) = B(i)\,,\; i = 2 \div n \tag{29}$$

$$P(n+1) = P_z \tag{30}$$

where

$$B(i) = -C(i)P0(i) - D(i)\,,\; i = 2 \div n \tag{31}$$

As the resulting algebraic system has a coefficient matrix of Jacobi type, i.e. tridiagonally, its solving becomes easy following the use of Thomas process [2]. Thus, we will firstly determine

$$a'(1) = 0\,;\; b'(1) = P_s \tag{32}$$

after which the following can be calculated for $i = 2 \div n$

$$a(i) = -\left[2 + C(i)\right];\; b(i) = -C(i)P0(i) - D(i) \tag{33}$$

$$a'(i) = \frac{1}{a(i) - a'(i-1)}\,;\; b'(i) = \frac{b(i) - b'(i-1)}{a(i) - a'(i-1)} \tag{34}$$

and finally, because $b(n+1) = P_z$, the solution of the system of equation is

$$P(n+1) = P_z\,;\; P(n-i+1) = b'(n-i+1) - a'(n-i+1)P(n-i+2) \tag{35}$$

5 CALCULUS EXAMPLE

We will consider a closed impermeable circular gas field having the outline radius of 200 m, 40 m thickness, 20% porosity and 10 mD permeability. The pressure of field gases is of 140 bar, and their temperature of 27 °C. The field is exploited by a well of 0,1 m radius. For pressure depending on gas viscosity and deviation factor, respectively, we accept the relations:

$$\mu(P) = 1 + 0{,}02P + 0{,}004P^2 \tag{36}$$

$$Z(P) = 1 - 0{,}062P + 0{,}004P^2 \tag{37}$$

experimentally determined from field curves.

We will consider a spatial pitch of digitization lattice of 0,01 a temporal pitch of 60 s, and 10^{-5} acceptable error of iterative calculus.

In diagram 1 the pressure curves at top of the well are showed for various values of flow extracted. It can be observed the extraction process becoming unstable for a certain flow value.

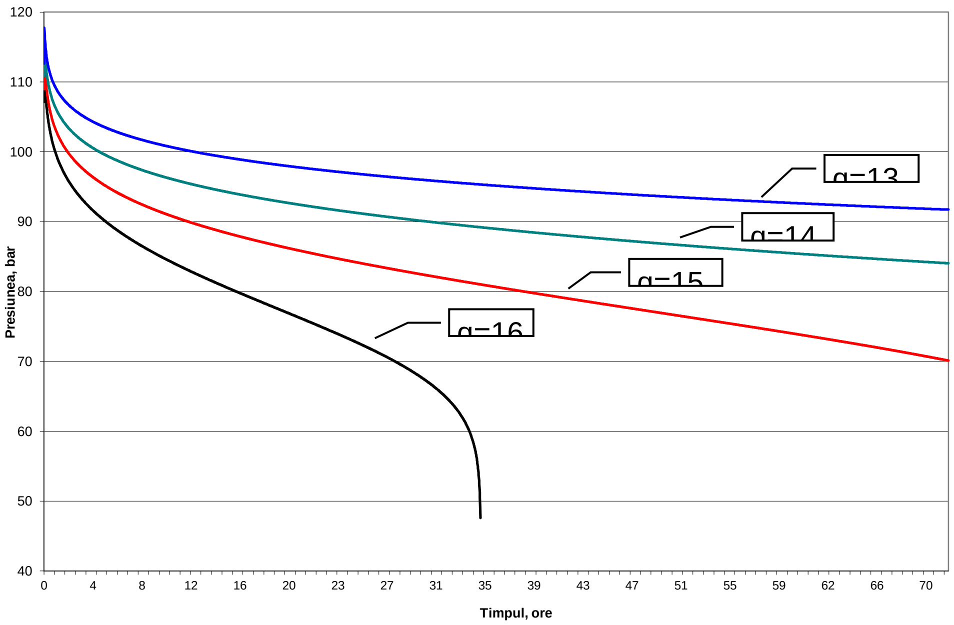

Diagram 1

ABSTRACT

It is regarded the isotherm movement of gases through a porous and isotropic medium towards a central well, taking into account the deviation from the perfect gases law and the viscosity pressure variation. The resulting model, completed with the specific limit conditions, is approached through a numerical method of solving and is applied to the wells through which the gas fields are exploited at constant flow. For the current exploitation rates of flow the variation curves of gas pressure at top of the well have been determined, thus resulting the flow maximum value after which the extraction process becomes unstable.

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